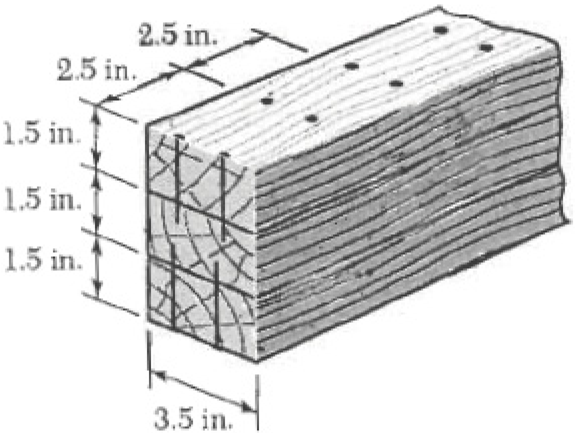
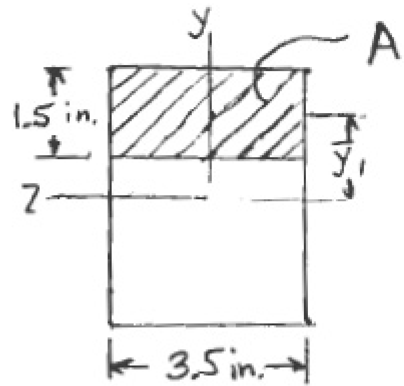
Problem 6.1



6.1 Three boards, each of 1.5 × 3.5-in. rectangular cross section, are nailed together to form a beam that is subjected to a vertical shear of 250 lb. Knowing that the spacing between each pair of nails is 2.5 in., determine the shearing force in each nail.

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(3.5)(4.5)^3 = 26.578 \text{ in}^4$$



$$A = (3.5)(1.5) = 5.25 \text{ in}^2$$

$$\bar{y}_1 = 1.5 \text{ in.}$$

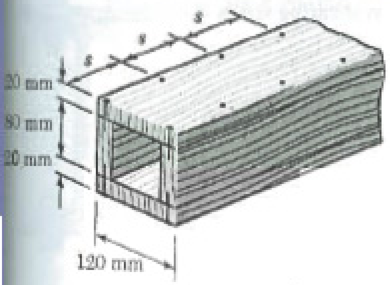
$$Q = A\bar{y}_1 = 7.875 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(250)(7.875)}{26.578} = 74.074 \text{ lb/in}$$

$$qs = 2F_{nail} \qquad F_{nail} = \frac{qs}{2} = \frac{(74.074)(2.5)}{2} = 92.6 \text{ lb.}$$

Problem 6.3

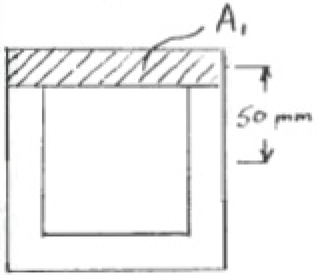
6.3 A square box beam is made of two 20 × 80-mm planks and two 20 × 120-mm planks nailed together as shown. Knowing that the spacing between the nails is s = 50 mm and that the allowable shearing force in each nail is 300 N, determine (*a*) the largest allowable vertical shear in the beam, (*b*) the corresponding maximum shearing stress in the beam.



$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(120)(120)^3 - \frac{1}{12}(80)(80)^3 = 13.8667 \times 10^6 \text{ mm}^4$$

$$= 13.8667 \times 10^{-6} \text{ m}^4$$



(a) $A_1 = (120)(20) = 2400 \text{ mm}^2$

$\bar{y}_1 = 50 \text{ mm}$

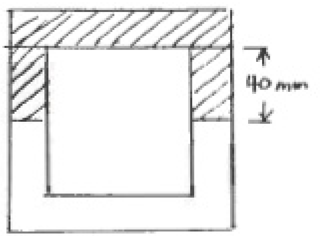
$Q_1 = A_1 \bar{y}_1 = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$

$$q_{all} = \frac{2F_{nail}}{s} = \frac{(2)(300)}{50 \times 10^{-3}} = 12 \times 10^3 \text{ N}$$

$$q = \frac{VQ}{I}$$

$$V = \frac{qI}{Q} = \frac{(12 \times 10^3)(13.8667 \times 10^{-6})}{120 \times 10^{-6}}$$

$$= 1.38667 \times 10^3 \text{ N} = 1.387 \text{ kN}$$



(b) $Q = Q_1 + (2)(20)(40)(20)$

$= 120 \times 10^3 + 32 \times 10^3 = 152 \times 10^3 \text{ mm}^3$

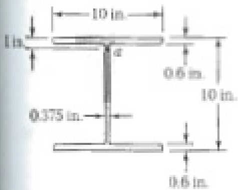
$= 152 \times 10^{-6} \text{ m}^3$

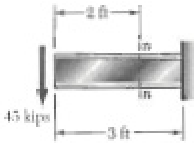
$$\tau_{max} = \frac{VQ}{It} = \frac{(1.38667 \times 10^3)(152 \times 10^{-6})}{(13.8667 \times 10^{-6})(2 \times 20 \times 10^{-3})}$$

$$= 380 \times 10^3 \text{ Pa} \qquad 380 \text{ kPa}$$

Problem 6.9

6.9 through 6.12 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.



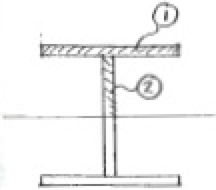


$V = 45$ kips

Part	A (in^2)	d (in.)	Ad^2 (in^4)	$\bar{I}$ (in^4)
Flange	6.00	4.7	132.54	0.18
Web	3.30	0	0	21.296
Flange	6.00	4.7	132.54	0.18
Σ			265.08	21.656

$$I = \Sigma Ad^2 + \Sigma \bar{I}$$
$$= 265.08 + 21.656$$
$$= 286.736 \text{ in}^4$$

(a)

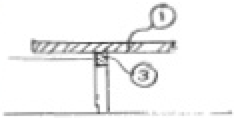


$$Q = A_1\bar{y}_1 + A_2\bar{y}_2$$
$$= (6.00)(4.7) + (0.375)(4.4)(2.2) = 31.83 \text{ in}^3$$

$t = 0.375$ in.

$$\tau_{max} = \frac{VQ}{It} = \frac{(45)(31.83)}{(286.736)(0.375)} = 13.32 \text{ ksi}$$

(b)



$$Q_a = A_1\bar{y}_1 + A_3\bar{y}_3$$
$$= (6.00)(4.7) + (0.375)(0.4)\left(\frac{4.4 + 4.0}{2}\right)$$
$$= 28.83 \text{ in}^3$$

$t = 0.375$ in.

$$\tau_a = \frac{VQ_a}{It} = \frac{(45)(28.83)}{(286.736)(0.375)} = 12.07 \text{ ksi}$$

Problem 6.26

6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear **V**. Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant *k* in the following expression for the maximum shearing stress

$$\tau_{max} = k\frac{V}{A}$$

where *A* is the cross-sectional area of the beam.

For a thin walled circular section, $A = 2\pi r_m t_m$

$J = A r_m^2 = 2\pi r_m^3 t_m$, $I = \frac{1}{2}J = \pi r_m^3 t_m$

For a semicircular arc, $\bar{y} = \frac{2r_m}{\pi}$

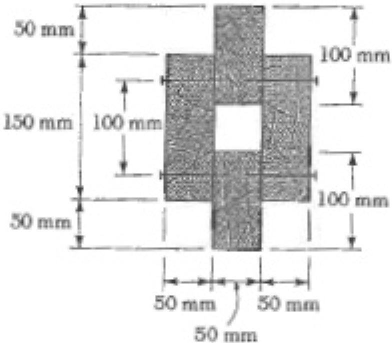
$A_s = \pi r_m t_m$ $Q = A_s\bar{y} = \pi r_m t_m \frac{2r_m}{\pi} = 2r_m^2 t_m$

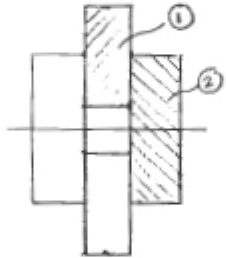
(a) $t = 2t_m$ at neutral axis where maximum occurs.

(b) $\tau_{max} = \frac{VQ}{It} = \frac{V(2r_m^2 t_m)}{(\pi r_m^3 t_m)(2t_m)} = \frac{V}{\pi r_m t_m} = \frac{2V}{A}$ $k = 2.00$

Problem 6.29

6.29 The built-up timber beam is subjected to a 6-kN vertical shear. Knowing that the longitudinal spacing of the nails is $s = 60$ mm and that each nail is 90 mm long, determine the shearing force in each nail.





$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2$$

$$= \frac{1}{12}(50)(100)^3 + (50)(100)(75)^2$$

$$= 32.292 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b h^3 = \frac{1}{12}(50)(150)^3$$

$$= 14.0625 \times 10^6 \text{ mm}^4$$

$$I = 2I_1 + 2I_2 = 92.71 \times 10^6 \text{ mm}^4 = 92.71 \times 10^{-6} \text{ m}^4$$

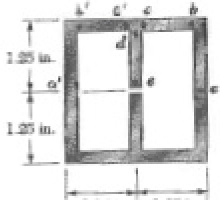
$$Q = Q_1 = A_1 \bar{y}_1 = (50)(100)(75) = 375 \times 10^3 \text{ mm}^3 = 375 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(6 \times 10^3)(375 \times 10^{-6})}{92.71 \times 10^{-6}} = 24.27 \times 10^3 \text{ N/m} \qquad s = 60 \text{ mm} = 60 \times 10^{-3} \text{ m}$$

$$2F_{nail} = qs \qquad F_{nail} = \frac{1}{2} qs = \frac{1}{2}(24.27 \times 10^3)(60 \times 10^{-3}) = 728 \text{ N}$$

Problem 6.37

6.37 and 6.38 The extruded beam shown has a uniform wall thickness of $\frac{1}{8}$ in. Knowing that the vertical shear in the beam is 2 kips, determine the shearing stress at each of the five points indicated.



$$I = \frac{1}{12}(2.50)(2.50)^3 - \frac{1}{12}(2.125)(2.25)^3 = 1.2382 \text{ in}^4$$

Add symmetric points c', b', and a'.

$$Q_e = 0$$

$$Q_d = (0.125)(1.125)\left(\frac{1.125}{2}\right) = 0.07910 \text{ in}^3 \qquad t_d = 0.125 \text{ in.}$$

$$Q_c = Q_e + (0.125)^2(1.1875) = 0.09765 \text{ in}^3 \qquad t_c = 0.25 \text{ in.}$$

$$Q_b = Q_c + (2)(1.0625)(0.125)(1.1875) = 0.41308 \text{ in}^3 \qquad t_b = 0.25 \text{ in.}$$

$$Q_a = Q_b + (2)(0.125)(1.25)\left(\frac{1.25}{2}\right) = 0.60839 \text{ in}^3 \qquad t_a = 0.25 \text{ in.}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(2)(0.60839)}{(1.2382)(0.25)} = 3.93 \text{ ksi}$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(2)(0.41308)}{(1.2382)(0.25)} = 2.67 \text{ ksi}$$

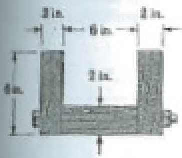
$$\tau_c = \frac{VQ_c}{It_c} = \frac{(2)(0.09765)}{(1.2382)(0.25)} = 0.63 \text{ ksi}$$

$$\tau_d = \frac{VQ_d}{It_d} = \frac{(2)(0.07910)}{(1.2382)(0.125)} = 1.02 \text{ ksi}$$

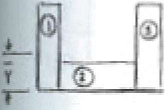
$$\tau_e = \frac{VQ_e}{It_e} = 0$$

Problem 6.43

6.43 A beam consists of three planks connected as shown by $\frac{3}{8}$-in.-diameter bolts spaced every 12 in. along the longitudinal axis of the beam. Knowing that the beam is subjected to a 2500-lb vertical shear, determine the average shearing stress in the bolts.



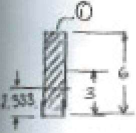
Locate neutral axis and compute moment of inertia.



Part	A (in²)	$\bar{y}$ (in.)	$A\bar{y}$ in³	d (in)	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
1	12	3	36	0.667	5.333	36
2	12	1	12	1.333	21.333	4
3	12	3	36	0.667	5.333	36
Σ	36		84		32	76

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{84}{36} = 2.333 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 108 \text{ in}^4$$



$$Q = A_1 \bar{y}_1 = (2)(6)(3 - 2.333) = 8 \text{ in}^3$$

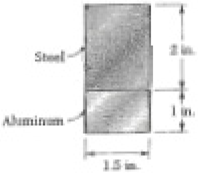
$$q = \frac{VQ}{I} = \frac{(2500)(8)}{108} = 185.2 \text{ lb/in}$$

$$F_{bolt} = qs = (185.2)(12) = 2.222 \times 10^3 \text{ lb.}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 = 0.1104 \text{ in}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{2.222 \times 10^3}{0.1104} = 20.1 \times 10^3 \text{ psi} = 20.1 \text{ ksi}$$

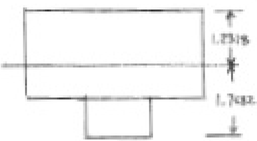
Problem 6.59



6.59 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum shearing stress in the beam. (*Hint*: Use the method indicated in Prob. 6.55.)

$n = 1$ in aluminum $\qquad n = \dfrac{29 \times 10^6 \text{ psi}}{10.6 \times 10^6 \text{ psi}} = 2.7358$ in steel

Part	nA (in²)	$\bar{y}$ (in)	$nA\bar{y}$ (in³)	d (in)	nAd^2 (in⁴)	$n\bar{I}$ (in⁴)
Steel	8.2074	2.0	16.4148	0.2318	0.4410	2.7358
Alum	1.5	0.5	0.75	1.2682	2.4125	0.1250
Σ	9.7074		17.1648		2.8535	2.8608



$$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma A} = \frac{17.1648}{9.7074} = 1.7682 \text{ in}$$

$$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.7143 \text{ in}^4$$

(a) At the bonded surface: $\quad Q = (1.5)(1.2682) = 1.9023 \text{ in}^3$

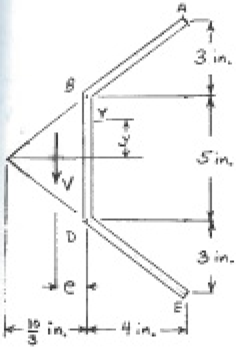
$$\tau = \frac{VQ}{It} = \frac{(4)(1.9023)}{(5.7143)(1.5)} = 0.888 \text{ ksi}$$

(b) At the neutral axis: $\quad Q = (2.7358)(1.5)(1.2318)\left(\frac{1.2318}{2}\right) = 3.1133 \text{ in}^3$

$$\tau_{max} = \frac{VQ}{It} = \frac{(4)(3.1133)}{(5.7143)(1.5)} = 1.453 \text{ ksi}$$

Problem 6.69

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



$L_{AB} = \sqrt{4^2 + 3^2} = 5$ in. $\qquad A_{AB} = 5t$

$I_{AB} = \frac{1}{12} A_{AB} h^2 + A_{AB} d^2 = \frac{1}{12}(5t)(3)^2 + (5t)(4)^2$

$= 83.75\ t$ in^4

$I_{BD} = \frac{1}{12}(t)(5)^3 = 10.417\ t$ in^4

$I = 2 I_{AB} + I_{BD} = 177.917\ t$ in.

In part BD: $\quad Q = Q_{AB} + Q_{BY}$

$Q = (5t)(4) + (2.5 - y)t(\frac{1}{2})(2.5 + y)$

$= 20t + 3.125t - \frac{1}{2}ty^2$

$= (23.125 - \frac{1}{2}y^2)t$

$\tau = \frac{VQ}{It} \qquad F_{BD} = \int \tau\, dA = \int_{-2.5}^{2.5} \frac{V(23.125 - \frac{1}{2}y^2)t}{It} \cdot t\, dy$

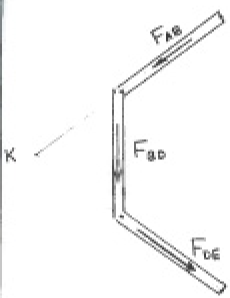
$= \frac{Vt}{I} \int_{-2.5}^{2.5} (23.125 - \frac{1}{2}y^2) dy = \frac{Vt}{I} \left[23.125y - \frac{1}{6}y^3 \right]_{-2.5}^{2.5}$

$= \frac{Vt}{I} \cdot 2 \left[(23.125)(2.5) - \frac{(2.5)^3}{6} \right] = \frac{Vt\,(110.417)}{177.917\ t}$

$= 0.62061\ V$

$\circlearrowright \Sigma M_K = \circlearrowright \Sigma M_K: \qquad -V(\frac{10}{3} - e) = -\frac{10}{3}(0.62061\ V)$

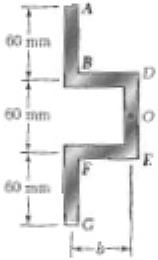
$e = \frac{10}{3}[1 - 0.62061] = 1.265$ in.



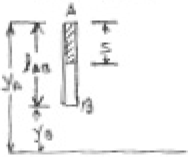
Note that the lines of action of F_{AB} and F_{DE} pass through point K. Thus, these forces have zero moment about point K.

Problem 6.78

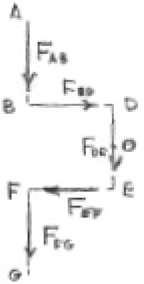
6.77 and 6.78 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.



Part AB:



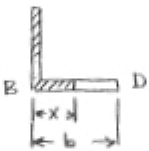
$$A(s) = ts$$
$$\bar{y}(s) = y_A - \tfrac{1}{2}s$$
$$Q(s) = A(s)\bar{y}(s) = t y_A s - \tfrac{1}{2}ts^2$$
$$q(s) = \frac{VQ(s)}{I} = \frac{Vt}{I}\left(y_A s - \tfrac{1}{2}s^2\right)$$
$$F_{AB} = \int_0^{l_{AB}} q(s)\,ds = \frac{Vt}{I}\left(\frac{y_A l_{AB}^2}{2} - \frac{l_{AB}^3}{6}\right)\downarrow$$



At B: $Q_B = t y_A l_{AB} - \tfrac{1}{2}t\, l_{AB}^2$

By symmetry, $F_{FG} = F_{AB}$

Part BD:



$$A(x) = tx$$
$$Q(x) = Q_B + y_B A(x) = Q_B + t y_B x$$
$$q(x) = \frac{VQ(x)}{I} = \frac{V}{I}(Q_B + t y_B x)$$
$$F_{BD} = \int_0^b q(x)\,dx = \frac{V}{I}\left(Q_B b + \tfrac{1}{2} t y_B b^2\right) \rightarrow$$

By symmetry, $F_{EF} = F_{BD}$

F_{DE} is not required, since its moment about is zero.

$$\circlearrowleft \Sigma M_O = 0: \quad b(F_{AB} + F_{FG}) - y_B F_{BD} + y_F F_{EF} = 0$$

$$2b F_{AB} - 2y_B F_{BD} = 0$$

$$2b \cdot \frac{Vt}{I}\left(\frac{y_A l_{AB}^2}{2} - \frac{l_{AB}^3}{6}\right) - 2y_B \frac{V}{I}\left(Q_B b + \tfrac{1}{2} t y_B b^2\right)$$

$$\frac{2Vt}{I}\left\{\tfrac{1}{2} y_A l_{AB}^2 - \tfrac{1}{6} l_{AB}^3\right\} b - \frac{2Vt}{I}\left\{\left(y_A l_{AB} - \tfrac{1}{2} l_{AB}^2\right) y_B b - \tfrac{1}{2} y_B^2 b^2\right\} = 0$$

Dividing by $\frac{2Vt}{I}$ and substituting numerical data,

$$\left\{\tfrac{1}{2}(90)(60)^2 - \tfrac{1}{6}(60)^3\right\} b - \left[(90)(60) - \tfrac{1}{2}(60)^2\right\}(30) b + \tfrac{1}{2}(30)^2 b^2 = 0$$

$$126\times10^3\, b - 108\times10^3\, b + 450\, b^2 = 0$$

$$18\times10^3\, b - 450\, b^2 = 0 \qquad b = 0 \text{ and } b = 40.0 \text{ mm}$$